Real-life scheduling problems are often very large-scale, so that the cost reduction effect by scheduling solvers is also very large. Thus, how closer the solver obtains to optimal solutions or conventional manmade solutions, is important. Also, from user's perspective, it is important to deal with variety of constraints flexibly. In this paper, we focus on variable depth search (VDS) which realizes intensive search by its larger search space represented by the long chain of simple neighborhood operations.

First, we provide a novel VDS algorithm for the multiple depot vehicle scheduling problem (MDVSP) which is a scheduling problem in which all the timetabled trips are assigned compatibly to any vehicle and are covered just once by any vehicle at minimum cost. In our algorithm, we utilize a basic neighborhood operation called "swap" which interchanges the latter part of the trip sequence for one vehicle and that for another vehicle. To realize VDS, we propose an algorithm called swap-path multiple backtracking algorithm (SMB) utilizing the fact that we can obtain a new assignment by a path. As a result, SMB traverses a larger search space strategically by regarding consecutive swap operations as a neighborhood.

Second, we introduce some theoretical results with detailed proofs. To begin with, we prove the fact that any assignment can be obtained from another assignment by
a finite number of swaps. From the viewpoint of the search tree induced by this fact, we can show that SMB is much faster than that for simple brute-force strategies. Then, we can show the case that \( n-1 \) swaps are required at most to obtain the nearest improved solution, where \( n \) is the number of trips. This result justifies our VDS approach which considers a larger search space. Moreover, we show a sufficient condition that SMB at threshold \( k \) can obtain an improved solution which is obtained by \( k \) swaps from the initial solution.

Third, we show an application of SMB algorithm into one of the most famous benchmark problems. To begin with, we show how to take SMB into a conventional framework for MDVSP with some enhancement techniques using pruning and tabu search heuristics. Computational results show that our method achieves an average of at least 20 % better results than other local search based methods, and that it exhibits the best early-time local search performance among state-of-the-art methods.

Fourth, we show another application of SMB algorithm into real-life railway rolling stock allocation problems (RSAP) at a railway company. First, we formulate RSAP as MDVSP with side constraints, and then propose a framework using iterative deepening and two-step backtracking approach to solve the problem. Computational results based on real constraints show that our approach can obtain an approximate solution near the optimum in shorter time than CPLEX.

Fifth, we propose a semiautomatic planning framework for creating the rolling stock allocation schedules using the SMB based engine. Our framework has two main features. First, users can easily register and adjust constraints flexibly through three kinds of constraint expressions. Second, users can obtain failure messages which identify the site of constraint violation accurately in failure cases. Our real-life trials show that our scheduler can provide results that are almost comparable to those obtained by experts faster than before.

The above-mentioned contributions show that the proposed approach is effective for large-scale real-life problems, and that it is independent of the particular choice of constraints. Therefore, it is possibly applicable for wide applications of other real-life problems based on MDVSP.